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Mechanical behavior and characterization of assembly in Timber Gridshells

GERBOISE project

Gridshell Elastique Réutilisable en BOIS longue duréE

Programme FEDER-FSE, opération MP0018193
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1- Introduction

The use of grid shell structures in architecture and structural engineering has risen in the past decade, yet fundamental research on the mechanics of such structures is lacking. Grid shells are long span structures comprised of a lattice of single layer members forming a curved surface. Grid shells can be made of a wide range of materials from steel to wood. They have potential to be used in readapting existing spaces or in new aesthetically pleasing structures. By studying their mechanics, engineers can be more effective at the schematic phase of design so that the potential of gridshells can be maximized.

This research project "GERBOISE" granted by the Région Occitanie consists of three partners, Quaternion, Terrell and INSA-LMDC. The first partner is Quaternion which focuses on the start-up, manufacturing and installing gridshells structure. The second is INSA-LMDC involved in the experimental procedures, measurements, load process and carrying out the tests with the European standards, also the implementation of the mechanical behavior of the connection in terms of Force – displacement relationship. The third partner Terrell is an engineering office which conducts with the implementation of the mechanical behavior of the connection in the entire structural model under Code-Aster.

2- Problem Statement

This research project conducts between the three partners mentioned above in order to study the behavior of the connection and characterize the ability of the connections to properly transfer the shear forces within the entire structure.

The problem statement of this research project is to identify and expect the global behavior of the assembly as it is under flexural behavior in both directions in the real case of the structure. In order to investigate the global behavior of the assembly, a preliminary experimental tests were done to collect the necessary information for the theoretical calculation, a complete beam was taken as a standard beam (bench mark) of the structure of the timber Gridshells in order to verify the expected capacity (force and displacement) of the beam under a certain load condition.

3- Analysis of previous experimental tests

Some theoretical calculations for a pervious experimental tests were investigated in order to compare the theoretical results with the experimental results. The experimental tests were investigated for 5 samples of timber beams without assembly. The total length of the tested beam 2.82 m, each beam was consisting of two wooden lattes connected by shear blocks each 60 cm. the average weight of the beams was around 4.44 kg, the beams were tested under a concentrated force in flexural test (3 points flexural test) as illustrated in figure 1 below:

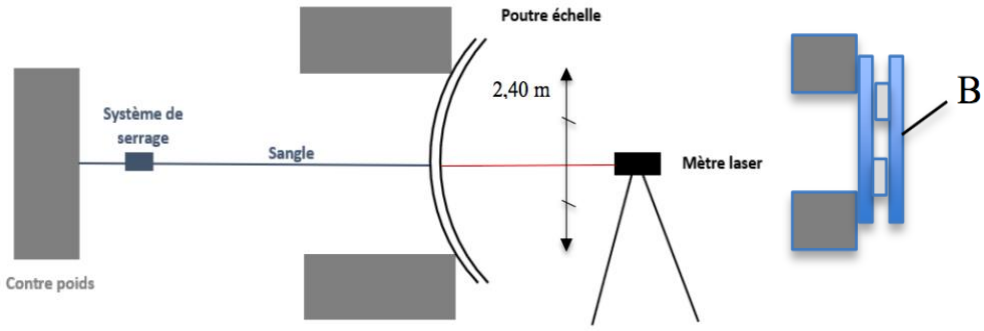


Figure 1: Wooden beam of the gridshells under flexural test (3 points).

The tested beams were in a vertical positions as shown above with a pull out cable that represents a horizontal force in order to apply the three points flexural test. The two end supports were totally free in order to permit the displacement in the horizontal direction. The Table 1 below summarizes the initial and final displacements of each beam. Obviously the average final displacement was around 45 cm.

Table 1: Displacement of wooden beams of the gridshells under flexural test.

Beam No.	Beam **(1)**	Beam **(2)**	Beam **(3)**	Beam **(4)**	Beam **(5)**
Initial displacement δ (cm)	**4.6**	**4.4**	**2.2**	**5**	**4.7**
Final displacement δ (cm)	**48**	**26**	**31**	**70**	**50**

- **Theoretical calculations**

The cross section of the wooden beam of the gridshells was consists of two lattes of thickness 12 mm and a width of 50 mm. A shear blocks of thickness 36 mm were fixed between the two lattes as shown in Figure 2 below.

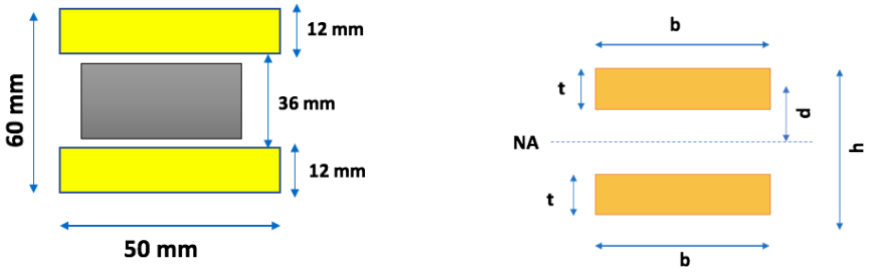


Figure 2: Cross section of the wooden beam of Gridshells of the previous tests.

The moment of inertia (I) and the maximum applied force (Fmax) were estimated theoretically according to the wood prosperities and the cross sectional area of the lattes.

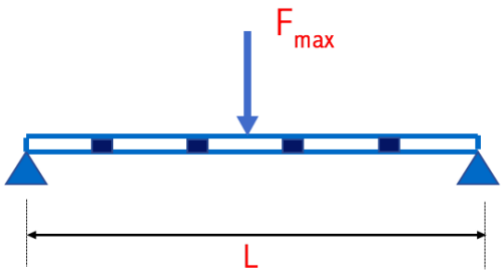
$$I = 2 \text{ x } \frac{bt^3}{12} + 2 \text{ x } b \text{ x } t\left(\frac{h}{2} - \frac{t}{2}\right)^2$$

$$I = 2 \text{ x } \frac{5\ (1.2)^3}{12} + \frac{5\ x\ 1.2}{2}(6 - 1.2)^2 = 1.44 + 69 = 70.5 \text{ cm}^4$$

$$M_{max} = \frac{F\ max\ \ L}{4}$$

$$\sigma_{max} = \frac{My}{I} \quad , \quad y = h/2 \quad then$$

$$F_{max} = \frac{8 \text{ x } \sigma \text{ x } I}{L \text{ x } h}$$



σ = 40 MPa , beam length L = 3.35 m , h= 6 cm

$$F_{max} = \frac{8 \text{ x } 4000 \text{ x } 70.5}{335 \text{ x } 6} = 1114 \text{ N}$$

The second group of the previous experimental test had the same cross section of the wooden beam of the gridshells consisting of two lattes of thickness 12 mm and a width of 50 mm. A shear blocks of thickness 36 mm were fixed between the two lattes.

Table 2: Displacement of wooden beams of the gridshells under flexural test.

Beam No.	Beam (1)	Beam (2)	Beam (3)	Beam(4)	Beam(5)
Weight(kg)	4.3	3.98	4.92	4.7	4.3
load (N/m)	12.6	11.7	14.4	13.8	12.6
δ exp 10^{-3} m	5.37	13.5	11	39.5	48

Experimental deflection

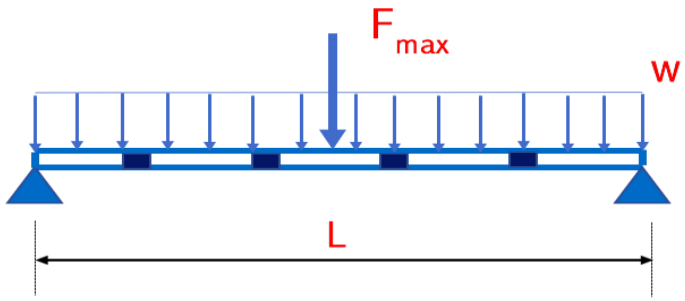
E = 10 GPa , L = 2.82 m , F = 30 N

$$\delta = \frac{FL^3}{48\,EI} \quad \text{(Concentrated load)}$$

$$\delta = \frac{5\,wL^4}{384\,EI} \quad \text{(Distributed load)}$$

$$\delta_{total} = \frac{FL^3}{48\,EI} + \frac{5\,wL^4}{384\,EI}$$

$$I_{total} = \frac{FL^3}{48\,E\delta} + \frac{5\,wL^4}{384\,E\delta}$$



4- Preliminary experimental calibration tests

(date: 14-15/10/2019)

Three wooden beams were tested at the laboratory in order to calibrate the main primary test and also to verify the total capacity of the beams (Force-displacement diagram). The beams were tested under the 3 points of flexural test. The end supports were fixed with clamps in order to allow the rotation and prevent the displacement in the horizontal direction.

According to the difficulties to apply the hydraulic jack as an applied force, a manual weights were applied gradually to the middle of the beam in order to represents the concentrated loads on the tested beams. As shown the results below, the failure mode of the first test was the space between the lattes and the shear blocks on the other hand the failure mode of the second test was at to top latte of the beam. The third test stopped once the load line distorted during the test. The results of each test in terms of force – displacement diagram are explained below:

Figure 3: Set up of the preliminary tests under flexural (3 points).

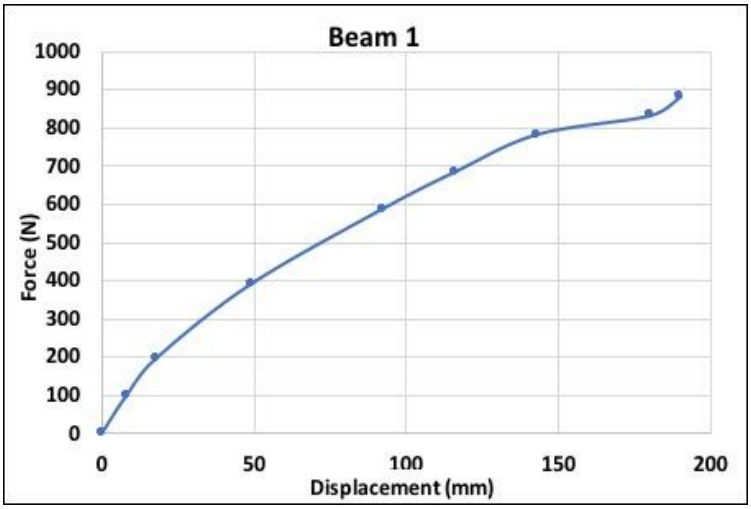


Figure 4: Force – displacement diagram of the first test.

Figure 5: Failure in Buckling.

During the first test, it was obvious that the maximum force applied to the wooden beam was around 900 (N). the mode of failure in the first test was the buckling occurred at the top wooden latte as shown in Figure 5. The second test followed the same load conditions as the previous one with a different mode of failure. As shown in figure 6 and 7 that the maximum applied force on the second beam was around 890 (N) with a debonding between the lower latte and the shear blocks as a failure mode.

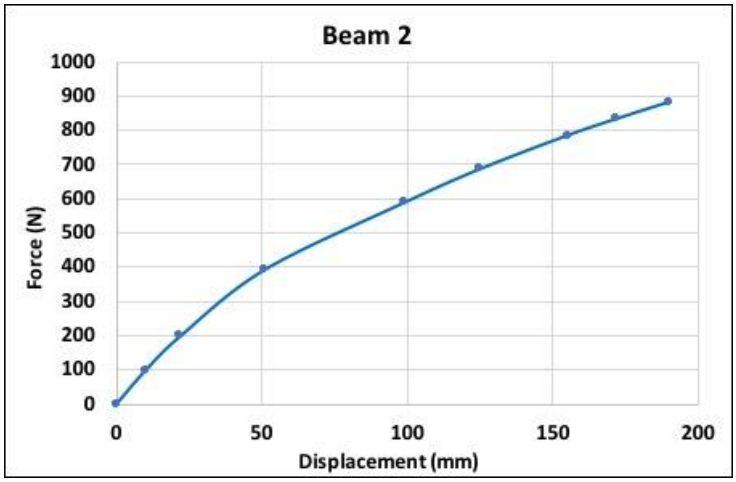


Figure 6: Force – displacement diagram of the first test.

Figure 7: Support failure.

The strengthening of support was prepared as a preparation for the third test in order to solve this failure mode and a void it for the next test. Screws were added to the ended support to prevent the debonding (to the lower wooden latte). The third test was tested with the same load conditions. The force – displacement diagram presented below a maximum applied force around 1KN before cutting the connector of the weight holder due to the heavy loads as shown in figure 9 below.

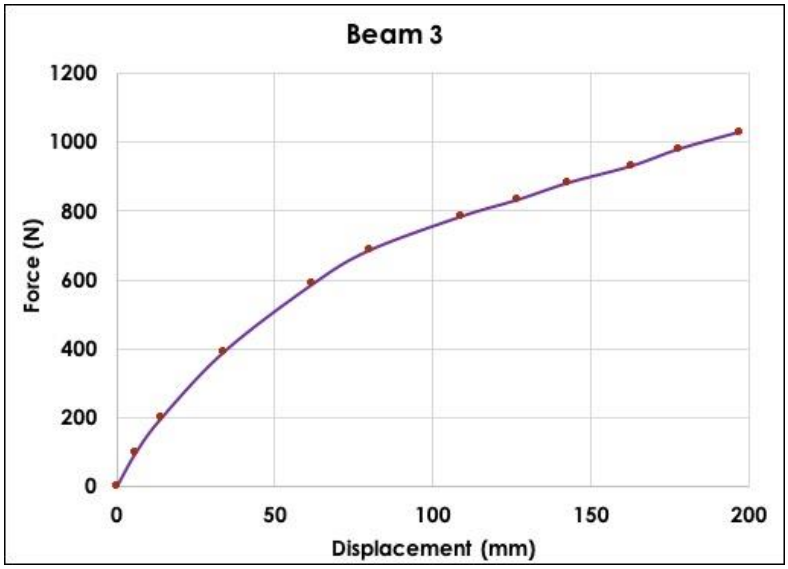


Figure 8: Force – displacement diagram of the third test.

Figure 9: Support failure.

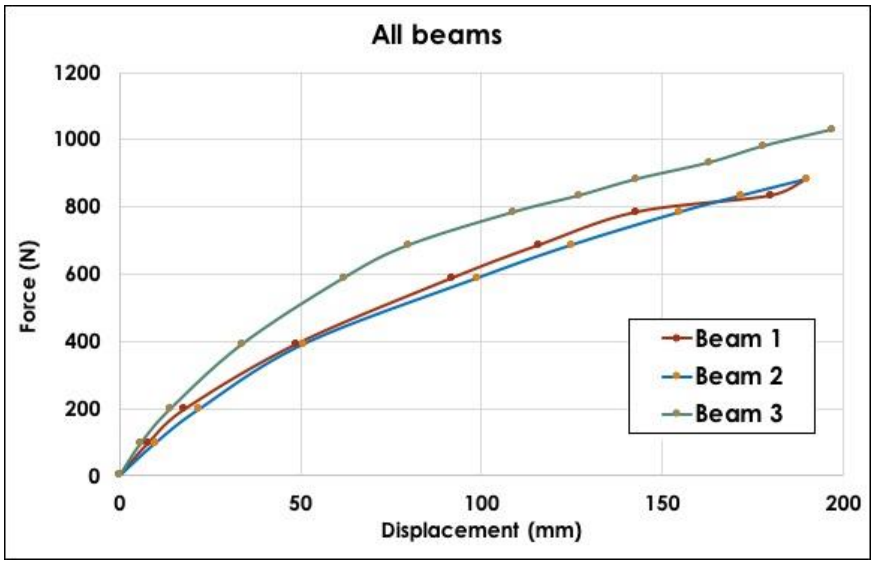


Figure 10: Force – displacement diagram of all tested beams.

5- Flexural test of wooden beam Group (1)

(date: 18-19-20/12/2019)

In order to investigate the mechanical behavior of the assembly under the flexural test of 4 point, a specific test up and prototype were prepared. One wooden beam was prepared with the steel connection as the case of the real structure. The strain gauges were fixed in both sides of the assembly. The gauges of the right side were fixed on the external steel surface (between wood and steel) and the strain gauges on the left side of the assembly were fixed on the internal steel surface considering the same positions of each gauge in both sides of the assembly.

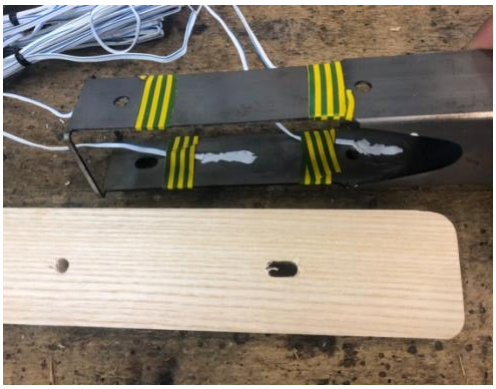


Figure 11: Strain gauges at the internal surfaces.

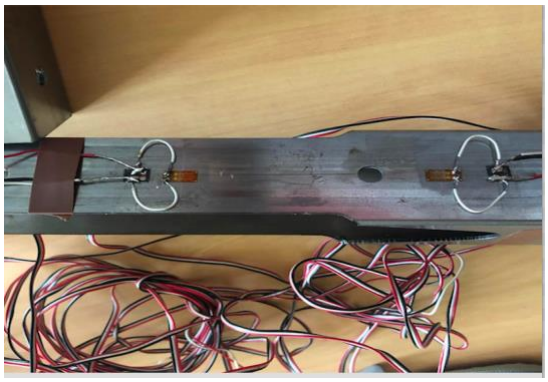


Figure 12: Strain gauges at the external surfaces.

Complete fabrication of the entire beam with the assembly were prepared for the test. The total length of the beam is around 2.4 m. The same test protocol was followed for this test but in this case the test was flexural test with four points in order to uniform the loads on the beam and avoiding the concentrated loads that cause a failure bucking on the top wooden latte as what occurred in the preliminary first test. The end supports were prepared in order to permit the rotation in both sides. The loads were applied manually as the preliminary tests

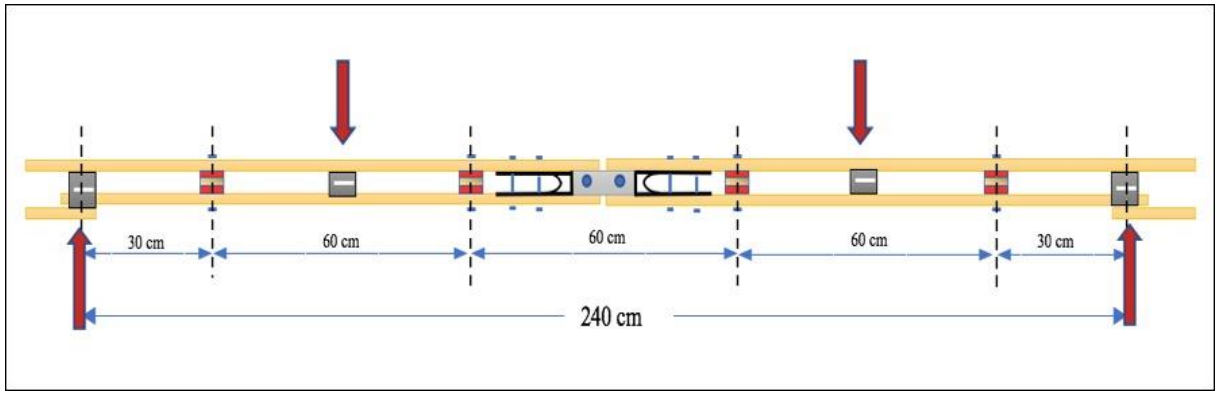


Figure 13: Four points flexural test of the entire wooden beam with assembly.

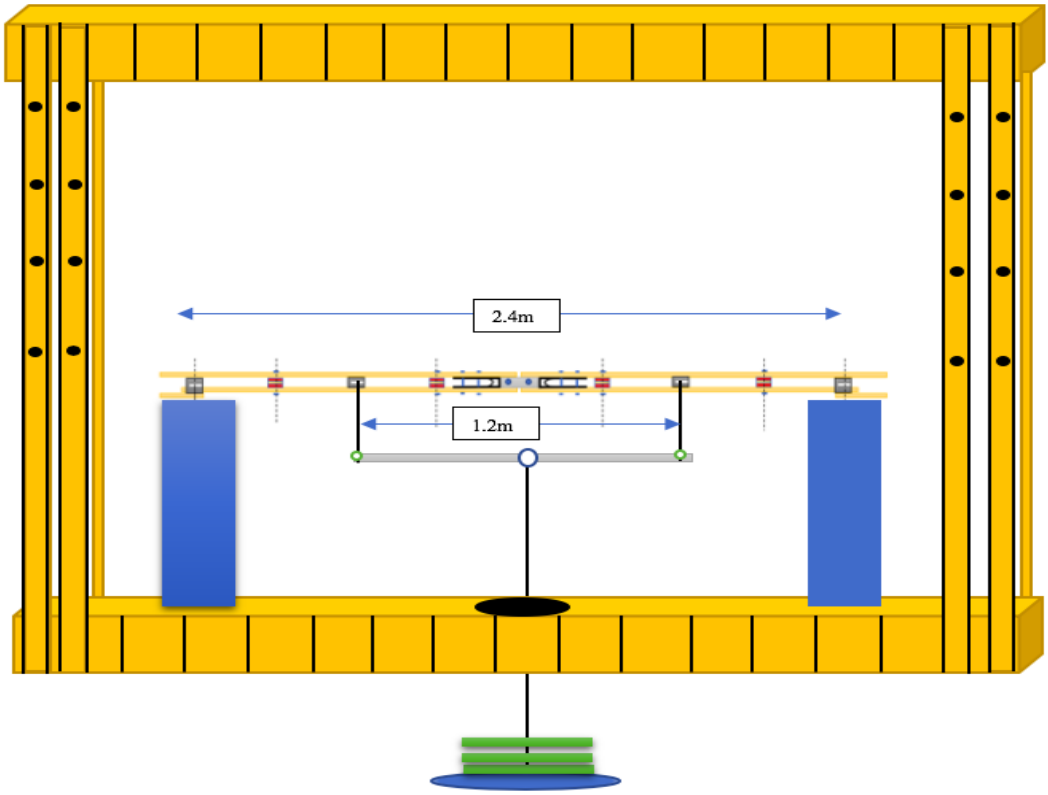


Figure 14: Test protocol of the entire wooden beam with assembly.

Due to the differences in the two pieces of the steel of the assembly, it was obvious that the entire wooden beam was not horizontally after the installation before the test, i.e there was an initial displacement before applying the loads. The laser device was fixed at the top center of the tested beam to measure the global displacement as shown in figure 14 below.

The force displacement diagram presented below describing the global behavior of the entire beam with assembly. Obviously, the global rigidity was increased due to the assembly. The first test were applied in loading and unloading, the same beam was loaded again with gradual loads

Figure 15: The initial displacement of the wooden beam with the assembly before the test.

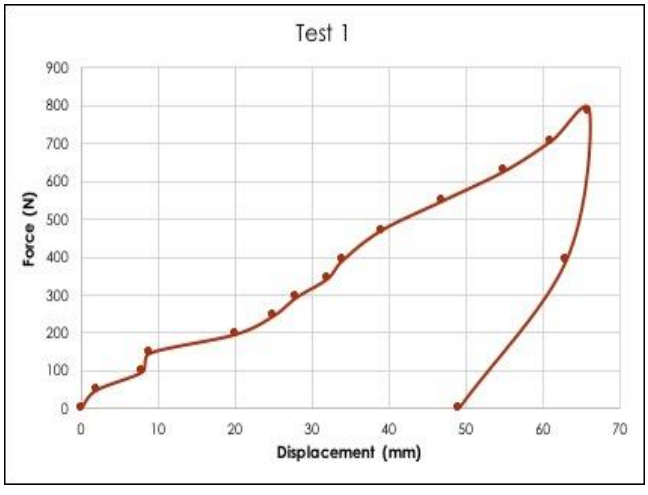


Figure 16: Force – displacement of the assembly test 1

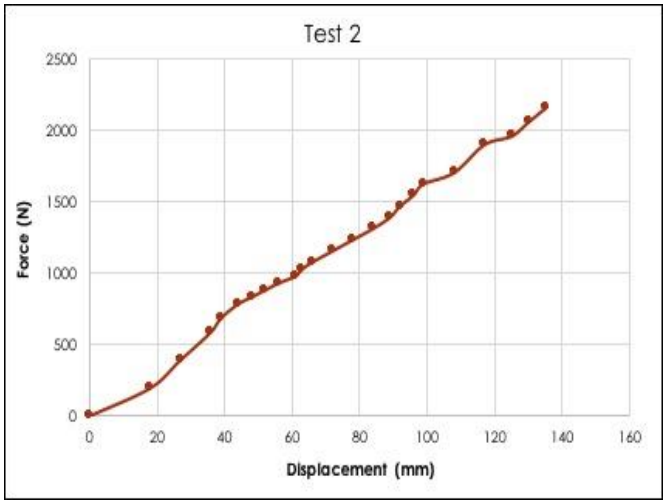
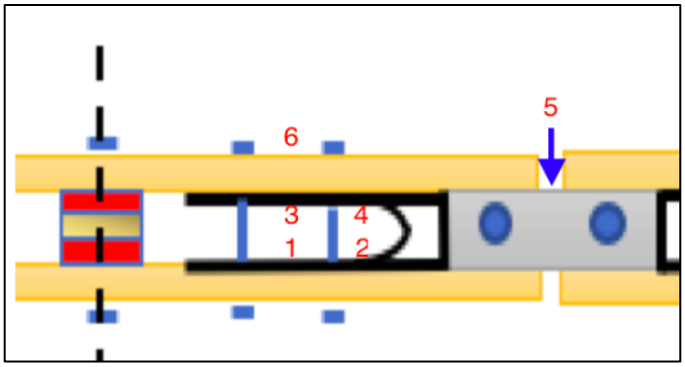


Figure 17: Force – displacement of the assembly test 2

The results of the strain gauges illustrated below presents the tension and compression zones of the left side of the assembly. Unfortunately, the gauges that fixed on the right side of the assembly were destroyed due to the friction between wood and steel pieces during the test. The lower steel plate of the assembly (gauge 1 and 2) were under tension zone. The top wooden layer of the beam (gauge 6) and the metallic piece at the middle (gauge 5) were under compression.



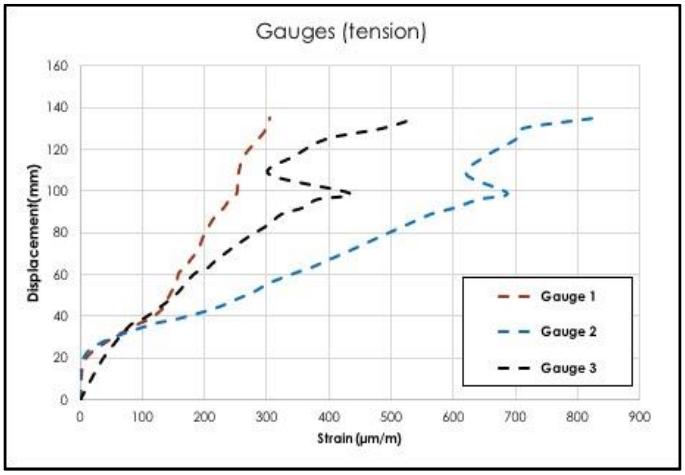


Figure 18: Strain gauges results in tension zone.

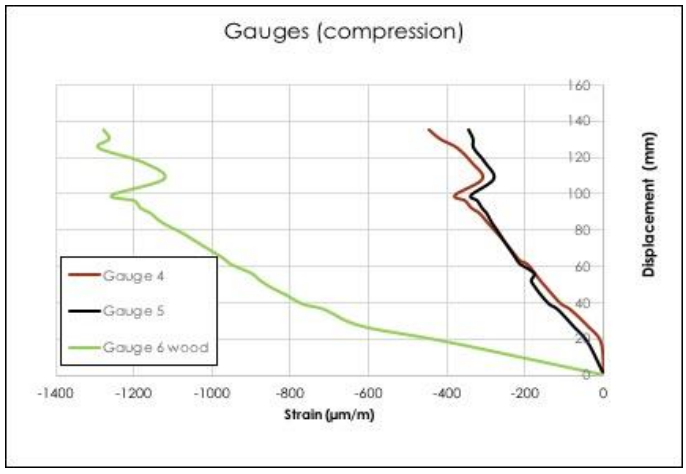
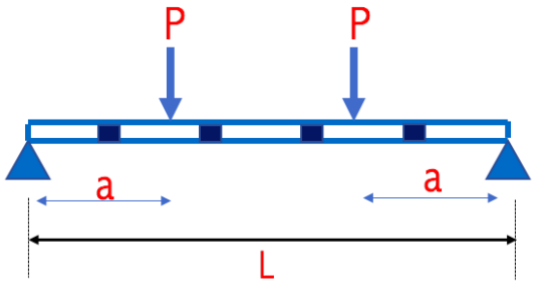


Figure 19: Strain gauges results in compression zone.

Experimental tests (4 point flexural test)

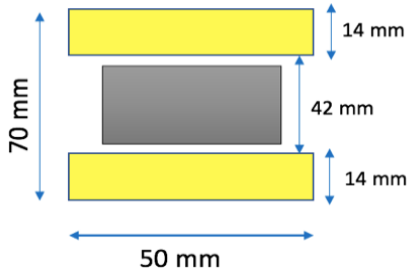
- **Theoretical calculation**



Length of beam (L) = 2.4m
Modulus of Elasticity (E) = 10 GPa
Bending moment at middle (M) = P. a

δ_{max} at (center) = $\frac{P.a}{24\,EI}(3L^2 - 4\,a^2)$

- Theoretical moment of inertia (I_{theo})

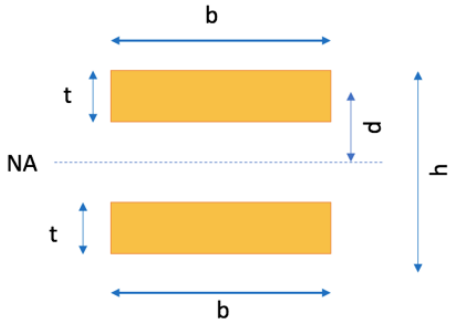


$I_{theo} = I_x + Ad^2$

$I_{theo} = 2 \times \frac{bt^3}{12} + 2 \times b \times t\,(\frac{h}{2} - \frac{t}{2})^2$

$I_{theo} = 2 \times \frac{5 \times 1.4^3}{12} + 5 \times 1.4\,(7 - 1.4)^2$

$I_{theo} = 112\ cm^4$



- Real moment of inertia (I_{real})

Data from the experimental results

Distance from load to the edge (a) = 600 mm , beam length (L) = 2400 mm

Elasticity modulus (E) = 10000 N/mm^2

δ_{max} at (center) = $\frac{P.a}{24\,EI}(3L^2 - 4\,a^2)$, P = 49 N at deflection 2mm then

$I_{real} = \frac{P.a}{24\,E\,\delta exp}(3L^2 - 4\,a^2)$

$I_{real} = \frac{49 \times 600}{24\,(10000)\,(2)}(3 \times 2400^2 - 4 \times 600^2)$ so $I_{real} = 97.02\ cm^4$

So there is a correction factor between the theoretical and real moment of inertia

$$I_{real} = K_I\ I_{theo}$$

Table 3: The relationship between the theoretical and experimental inertia of the beam.

Force F (N)	Load P (N)	δ_{exp} (mm)	$I_{real(exp)}$ cm^4	$K_I = I_{real} / I_{theo}$
0	0	0	0	
98	49	2	97.02	0.86625
196	98	4	97.02	0.86625
294	148	6	97.02	0.86625
392	196	9	86.24	0.7700
490	245	12	80.85	0.72187
588	294	16	72.77	0.64973
686	343	20	67.91	0.60634
764	382	23	65.77	0.58723
843	421.5	26	64.20	0.57321
922	461	31	58.89	0.52580
1000	500	36	55.00	0.49107
1079	539.5	41	52.11	0.46526
1157	578.5	47	48.74	0.43517
1177	588.5	51	45.70	0.40803
1275	637.5	57	44.29	0.39544
1373	686.5	64	42.48	0.37928
1471	735.5	71	41.02	0.36625
1569	784.5	80	38.83	0.34669

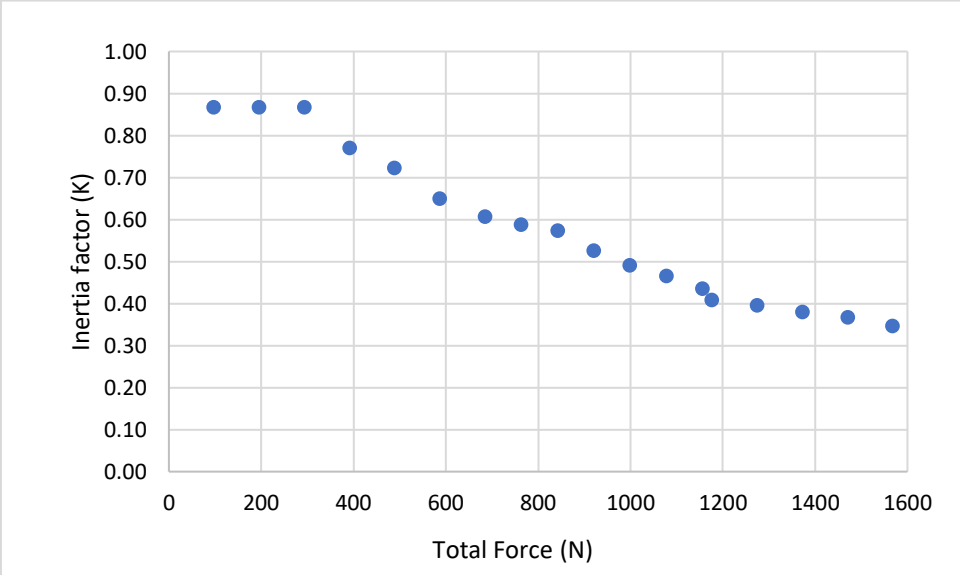


Figure 20: Inertia factor for the entire beam.

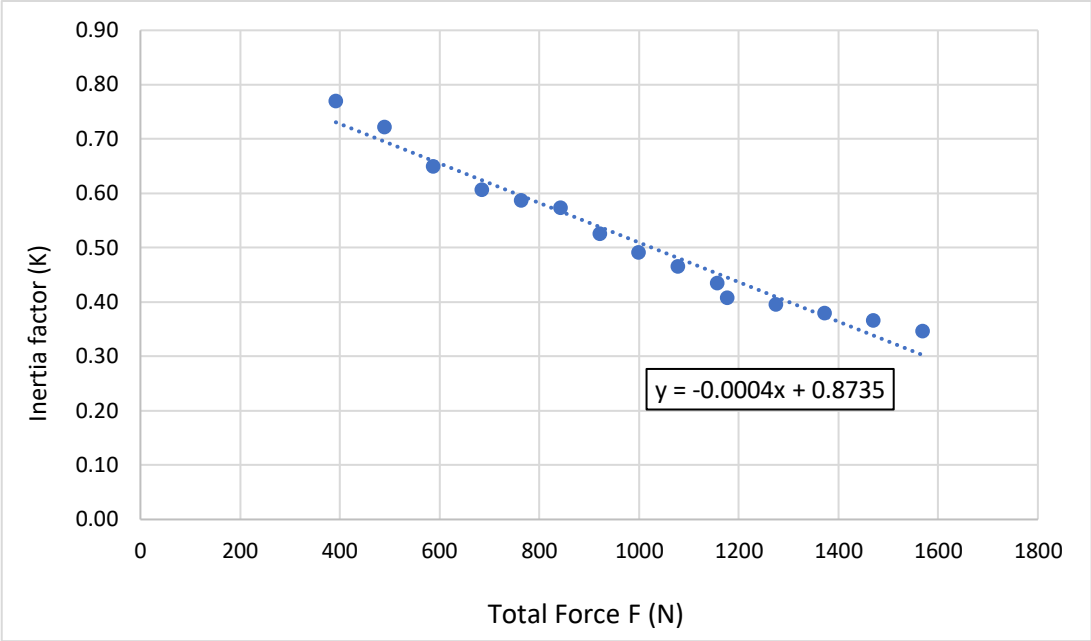


Figure 21: Inertia as a function of the load.

So it mean that the inertia is a function of the force

- ***First Zone (Elastic)***

$K_I = 0.866$ ⟹ $I_{real} = 0.866\ I_{theo}$

- ***Second Zone***

$$K_I = 0.8735 - 0.0004\,(F)$$

$$K_I = 0.8735 - 0.0008\,(P)$$

So now the real deflection can be reformulate according to the correction factor of the moment of inertia of the cross section as below:

- ***First Zone (Elastic)***

$$\delta_{real} = \frac{P.a}{24\,E\,(0.866\,I)}\,(3L^2 - 4\,a^2)$$

$$\delta_{real} = \frac{P.a}{20.8\,EI}\,(3L^2 - 4\,a^2)$$

$$\delta_{real} = \frac{P.a}{24\,EI(0.8735-0.0008P)} \quad (3L^2 - 4\,a^2)$$

$$\boxed{\delta_{real} = \frac{1}{EI}\,\frac{P.a}{(20.96 - 0.0192\,P)} \quad (3L^2 - 4\,a^2)}$$

- Beam slope calculation (rotation)

X1 (centroid of A1) = 60 (2/3) = 40 cm

X2 (centroid of A2) = 60 +30 = 90 cm

M = P × a

F_{max} = 1569 N , P_{max} = 784.5 N

A1 = (1/2) × 0.6 × 784.5 × 0.6

A1 = 141.2 N.m^2

A2 = 0.6 × 0.6 × 784.5 = 0.2826

A2 = 282.4 N.m^2

The slope at the support is given by the area

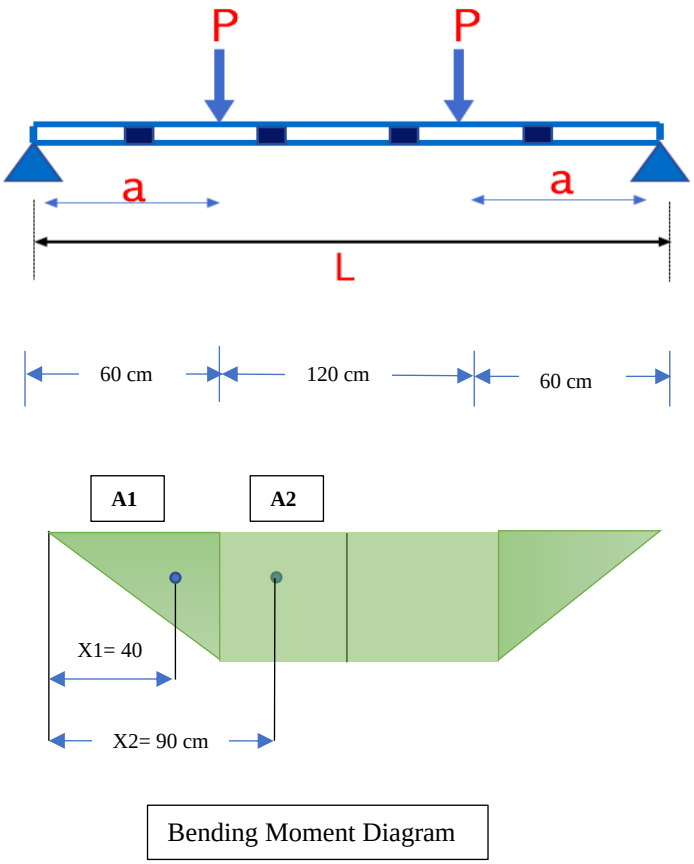
Of the moment diagram between A and C

Divided by EI

E = 10 GPa , I_{real} = 38.83 cm^4

$\theta_A = (A_1 + A_2)\ /EI$

$$\theta A = \frac{(141.2 + 282.4)}{EI} = \frac{(423.6)}{3883} = 6.3\,^\circ$$



Bending Moment Diagram

Table 4: Theoretical and Experimental beam rotation.

Force F (N)	Load P (N)	$I_{real(exp)}$ cm^4	Moment (N.m)	Rotation θ_{exp}	Rotation θ_{theo} (degree)
0	0	0	0	0.3819	0
98	49	97.02	29.4	0.4774	0.1563
196	98	97.02	58.8	0.5729	0.3125
294	148	97.02	88.2	0.6684	0.4688
392	196	86.24	117.6	0.7638	0.7032
490	245	80.85	147.0	0.9548	0.9376
588	294	72.77	176.4	1.1457	1.2500
686	343	67.91	205.8	1.3366	1.5627
764	382	65.77	229.2	1.4320	1.7970
843	421.5	64.20	252.9	1.6229	2.0313
922	461	58.89	276.6	1.8137	2.422
1000	500	55.00	300	2.0999	2.8127
1079	539.5	52.11	323.7	2.2906	3.2032
1157	578.5	48.74	347.1	2.5765	3.6723
1177	588.5	45.70	353.1	2.7671	3.9843
1275	637.5	44.29	382.5	3.0528	4.4534
1373	686.5	42.48	411.9	3.4336	5.0000
1471	735.5	41.02	441.3	3.71899	5.5476
1569	784.5	38.83	470.7	4.09918	6.2509

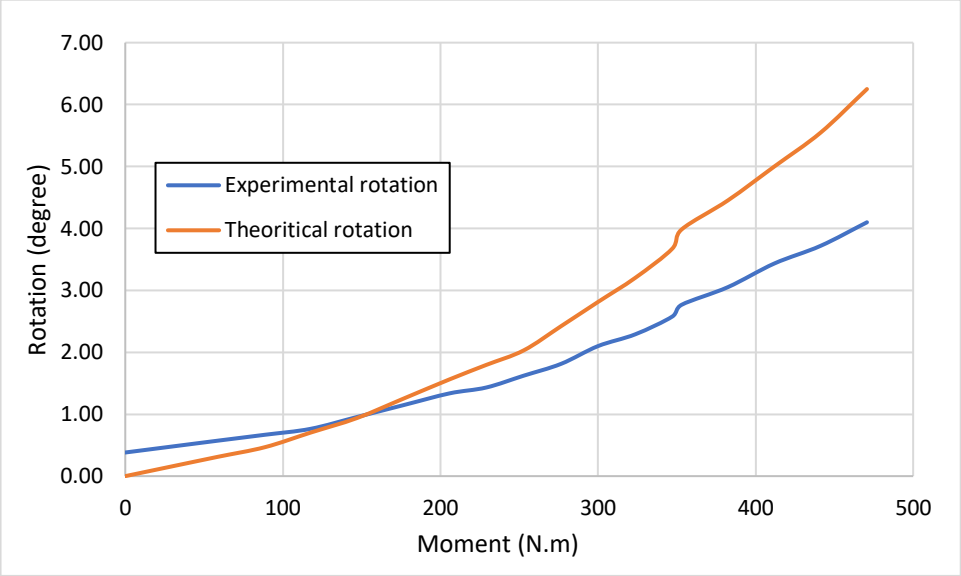


Figure 22: Experimental and Theoretical beam rotation of the entire beam.

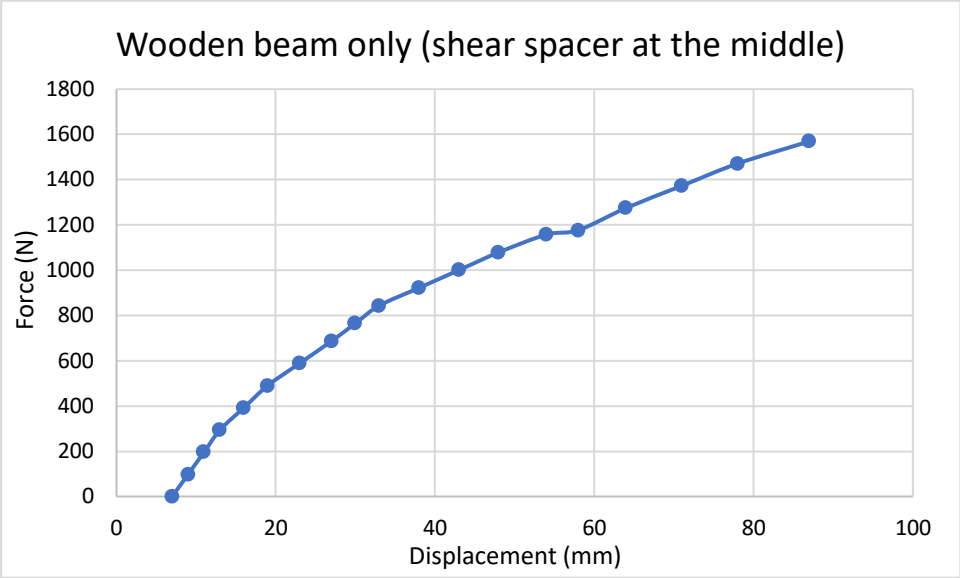


Figure 23: Force displacement diagram of wooden beam only.

During the test of 4 point flexural test for the wooden beam only (without assembly), it was obvious the initial displacement of the beam before loading around 7 mm. The global behavior of the beam is elastic (Figure 21). The measurements of the sliding between the two lattes were taken during the test and with different values of loads as illustrated in the table 5 below. According to the results of the sliding between the right side and left side of the beam, it was noted that the total sliding on the left side of the beam (left support) was around 5mm, on the other hand, the total siding on the right side of the beam (right support) was around 4.5 mm. the measurements of the sliding between the two lattes on the supports was an excellent sign to explain that the global behavior of the wooden beam during the load is symmetrical on both sides.

Table 5: Experimental sliding between the top and bottom lattes in wooden beam only.

Left side of the beam			Right side of the beam		
Weight (kg)	Force F (N)	Sliding (mm)	Weight (kg)	Force F (N)	Sliding (mm)
94	922	2	94	922	1
118	1157	1	118	1157	1
140	1373	1	140	1373	1
160	1569	1	160	1569	1.5
Total siding	5 mm		Total sliding	4.5 mm	

6- Flexural tests of wooden beams Group (2)

(date: 17-18/02/2020 and 19-20/05/2020)

The second group of the experimental tests was consists of testing the wooden beam only (Test No. 1 and Test No. 2) and the wooden beam with assembly (Test No. 3 and Test No. 4). The two wooden beams were prepared at the laboratory following the same protocol and test set up as the previous tests. The total length of the wooden beams was 2.4 meter, the shear wedge were fixed each 30 cm for the entire wooden beam, also one shear wedge was fixed, without nails, at the middle to avoid the buckling of the latte during the test as illustrated in Figure 23 .

The same wooden lattes of the beam in test 1 and test 2 were used again for the Test 3 and 4 after installing the steel assembly in each beam. The idea was to make a real representation of the real structure as the concept of the Gridshells multi- uses in the reality.

Another wooden beam was fixed with the main test to transfer and represent the two concentrated forces on the main beam as illustrated in Figure 23 below. The weights were added gradually and the deflection of the was registered in each value of the weight. Three laser machines L1, L2 and L3 were installed at the top side in the laboratory to investigate the deflection at three positions, The first laser machine L1 was fixed to investigate the deflection at the middle of the beam, the second laser L2 was fixed after 30 cm from the first one and the third laser L3 was fixed after 60 cm from the L1.

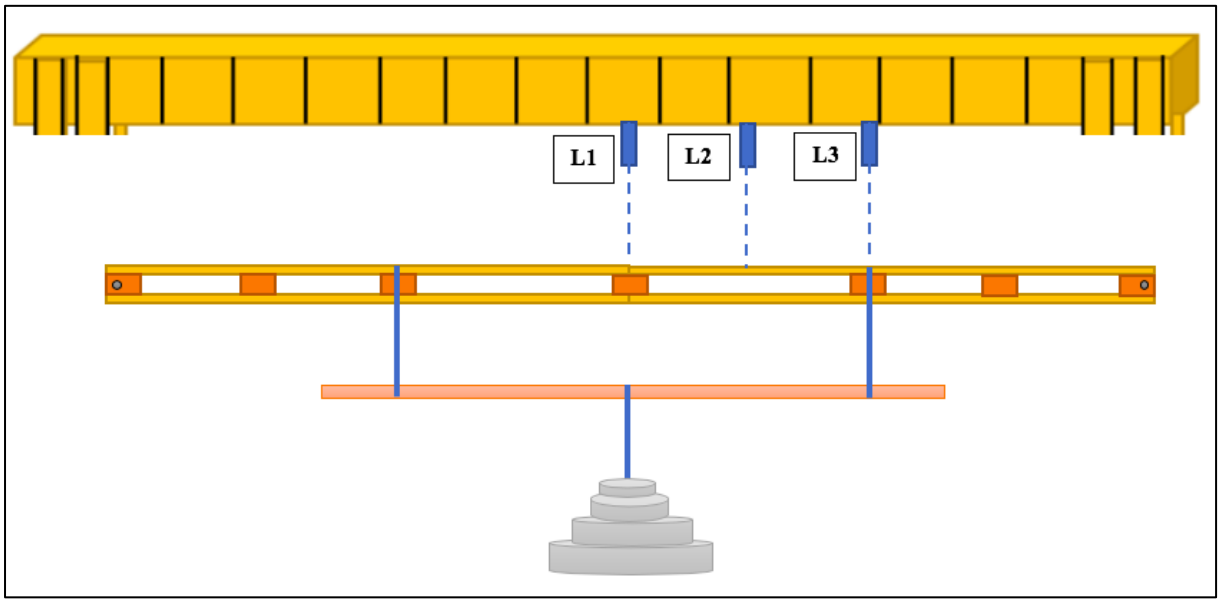


Figure 24: The setup of the wooden beam only for 4 point flexural test (without assembly).

- *Flexural test 4 points for wooden beams only*

(date: 17-18/02/2020)

As illustrated in Figure 25 below, the wooden beam without assembly were fixed as a simply supported beam with a free rotation at the supports. The three laser devices were installed to measure the deflection at three points at the beam. The initial deflection of the beam at the middle and before applying the loads was 7 mm for the first Test and 10 mm for the second test (see Figure 27 and Figure 28). The global behavior of the wooden beams were elastic. The maximum applied force was 1600 N with an average deflection of 80 mm at the middle of the beam.

Figure 25: Wooden beams without assembly in the test.

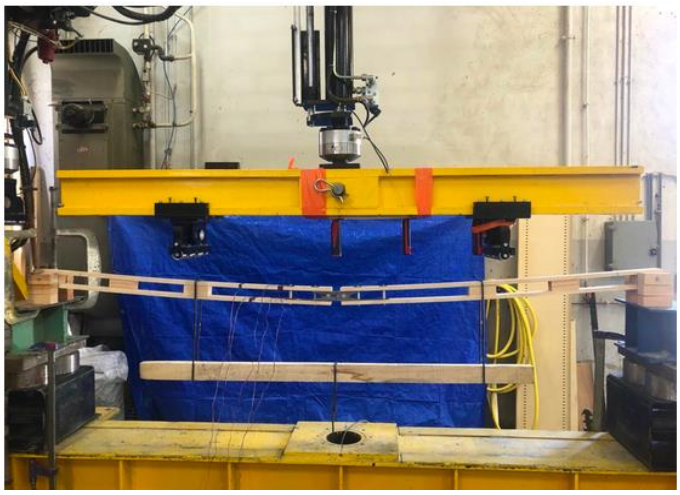


Figure 26: Wooden beam with assembly in the test.

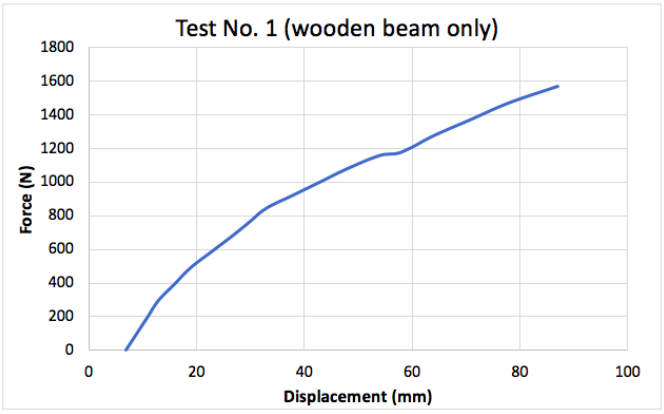


Figure 27: Force-displacement of wooden beam only test 1

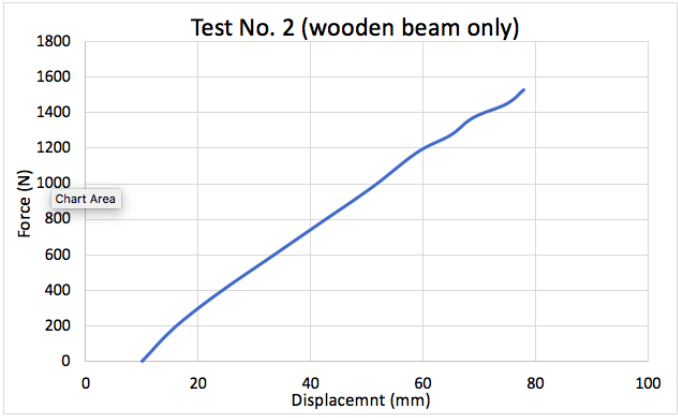


Figure 28: Force-displacement of wooden beam only test 2

- *Flexural test 4 points for wooden beams with assembly*

(date: 19-20/05/2020)

The same setup and test protocol were followed for the flexural tests of wooden beams with assembly. The same positions of laser devices L1,L2 and L3 to measure the global deflection of the entire beam. The beams were installed as a simply supported beams with ability of rotation at the end of each support. The strain gauges were installed also on different positions on the assembly, 4 gauges were fixed on the top and bottom parts of the steel part of the assembly as illustrated in Figure 29 below, and one gauge was installed on the top wooden part of the beam. The objective of gauges installation was to compare the behavior of each part of the assembly against shear forces and how the steel parts of assembly will deform comparing to the wooden part of the beam.

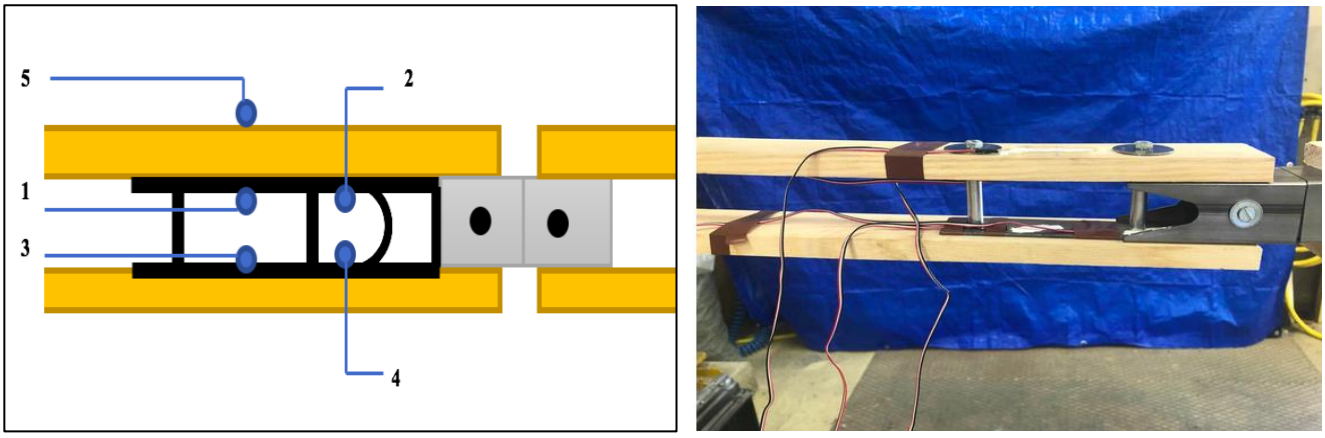


Figure 29: Positions of strain gauges on the assembly and top wooden latte.

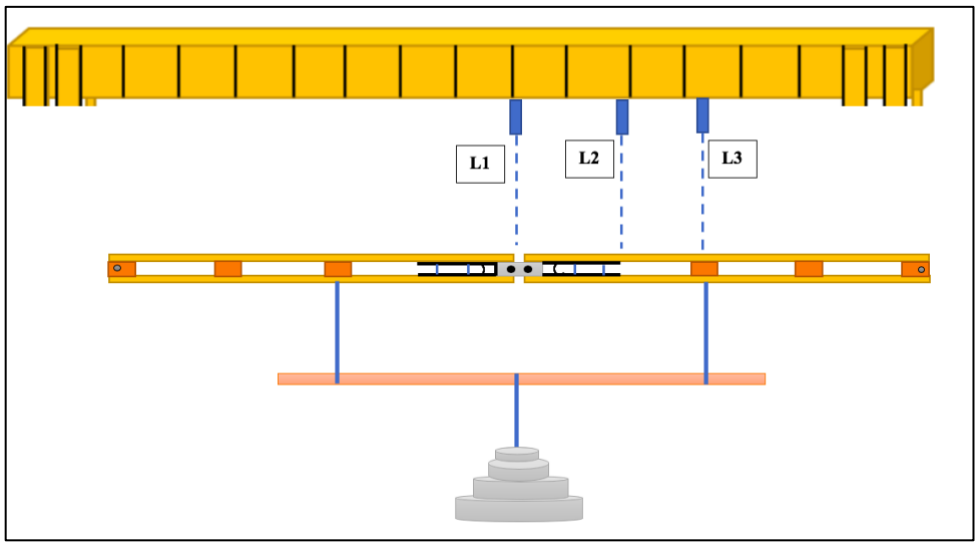


Figure 30: The setup of the wooden beam with assembly for 4 point flexural test.

The results in Figure 31 and Figure 32 represent the force displacement of wooden beams with assembly for Test 3 and Test 4 respectively. It was obvious that for the test no. 3 of wooden beam with assembly that the initial deflection before loading around 33 mm and for the test no.4 around 53 mm. The maximum applied force for both beams was around 1600 N with an average deflection 120 mm.

The strain gauges results were illustrated in Figure 33 and figure 34. With the assumption that the steel assembly is working perfectly with the wooden latte, then logically the top surface of the steel assembly with the top side of the wooden latte should be under compression in loaded case, on the other hand, the bottom surface of the wooden latte and the bottom surface of the assembly should be under tension.

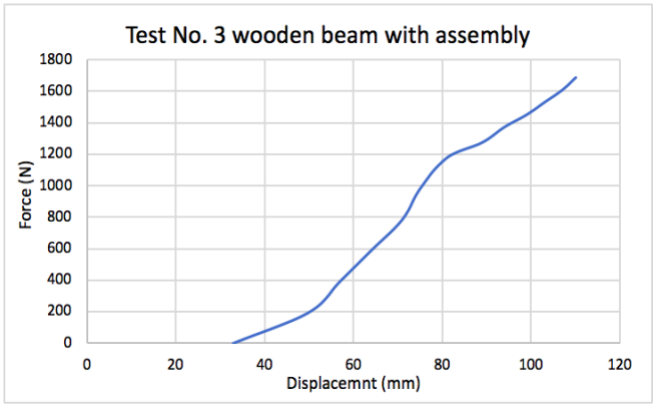


Figure 31: Force-displacement of wooden beam with assembly test 3.

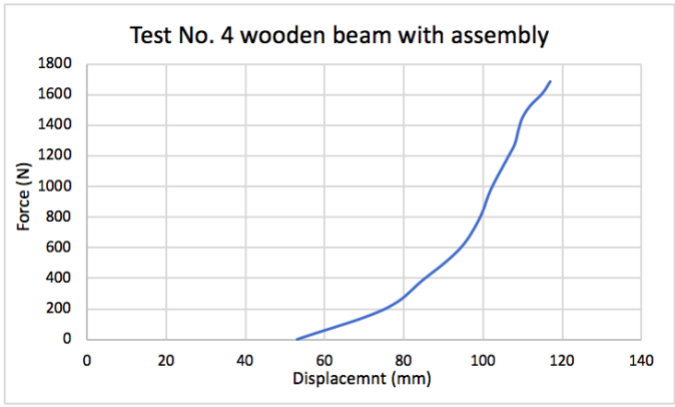


Figure 32: Force-displacement of wooden beam with assembly test 4.

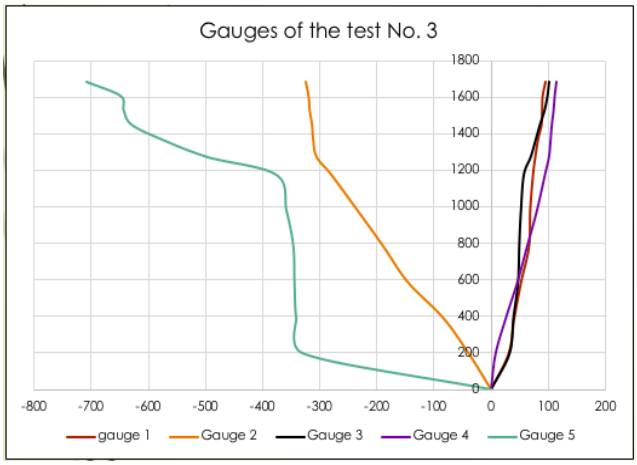


Figure 33: Strain gauges results of test 3.

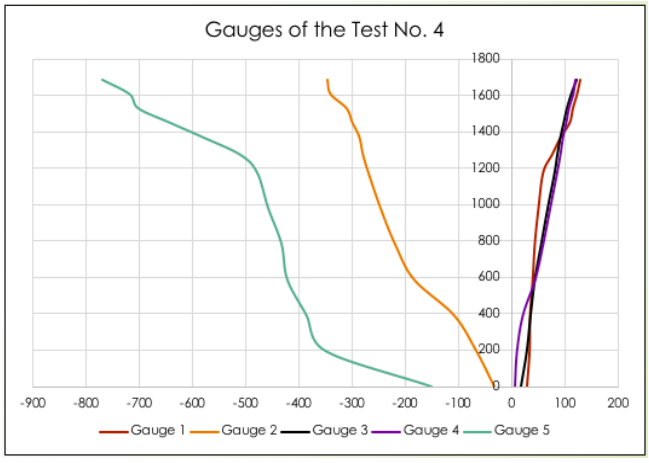


Figure 34: Strain gauges results of test 4.

According to the test results of the strain gauges, it was obvious that the bottom part of the assembly (Gauge 3 and 4) was under tension. The top part of the assembly was not completely under compression. The assembly part at Gauge no.2 was working with the top part of the wooden latte in gauge no. 5 under compression , on the other hand, the gauge no. 1 was under tension. This results explain that the spacers between the two parts of the assembly were not well designed. The left spacer normally was in making pressure on the top part of the assembly which reflect a tension behavior on this part.

The Figure 35 below shows the comparison in behavior between the beams without assembly and the beams with assembly. The results show that the wooden beams without assembly have approximately the same behavior at the beginning of the test (at 20 mm of deflection). The results show also that the beams with assembly have different behavior, these deference refer to the manufacture of the assembly. As noticed during the test, the spacer between the bolts were not well designed regarding , also the internal tube in steel was not in well contact with the assembly, this means that in each time of using the same beam will show a different behavior.

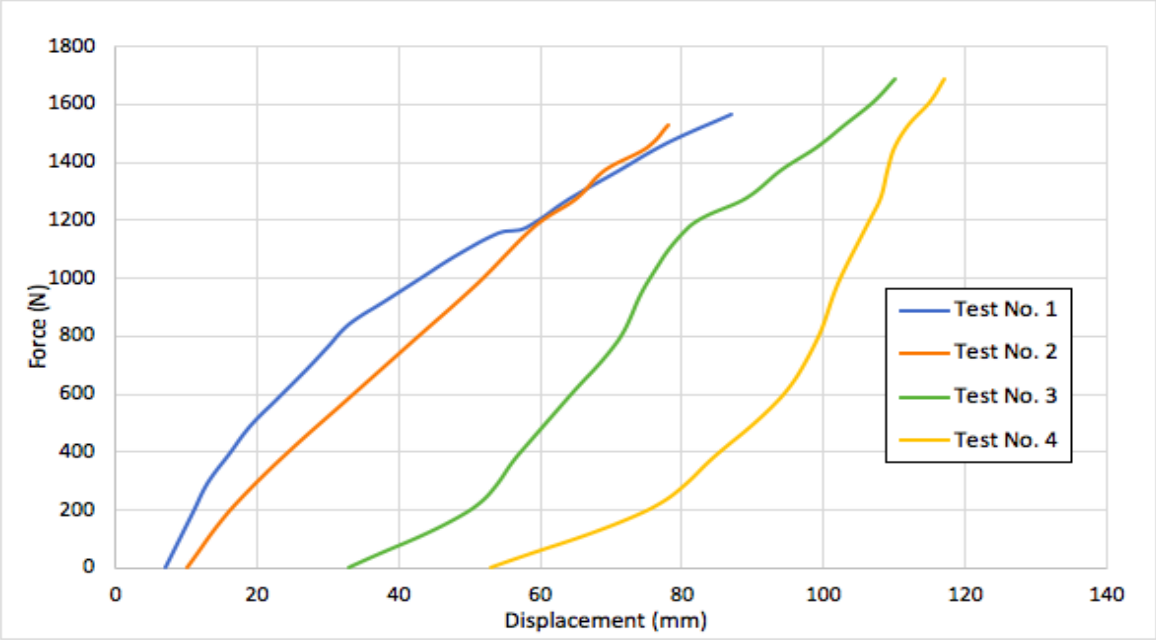


Figure 35: flexural tests of wooden beams with and without assembly.

7- Rigidity of beam parties

In order to characterize the total rigidity of the entire beam with the assembly, it was necessary to identify the rigidity of each part of the beam at the laboratory. As illustrated in Figure 36 below, The entire beam consists of shear wedges(K1), assembly of wood-steel connection (K2) and Assembly of steel-steel connection (K3).

In loaded state of the beam, the shear wedges will be under the shear forces transmit between the two wooden lattes. The assembly of steel-steel connection at the middle of the beam will be under flexural rotation. The assembly wood-steel connection will be under shear effect. In order to identify experimentally the rigidity of each part, small samples of each part were prepared.

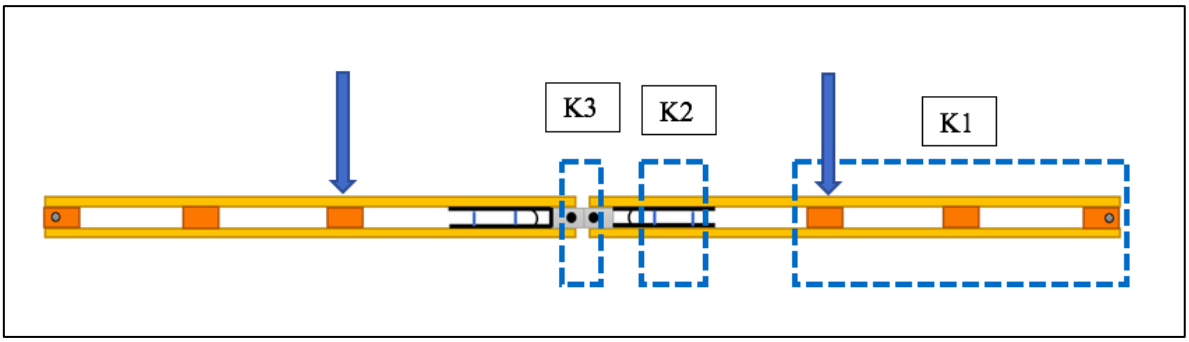


Figure 36: Beam parts of rigidity.

- *Shear wedges test*

(date: 07/07/2020)

Three samples of wood to wood connection were prepared at the laboratory. In each sample 4 screws of 3 mm were used (two screws) in each side. Two displacement sensors were fixed at the two edges in the sample to investigate the displacement between the wooden latte and the shear wedge, the total capacity of each sensor was 10 mm (Figure 37). A wooden foundation was fixed at the end of each sample for ensuring the stability of the sample during the test as illustrated in the figure 37 below. A speed test 2 mm / min was applied at each sample with the compression machine at the laboratory. The force displacement diagram was registered for each sample. According to tests results in Figure 39 it was obvious that the first zone of the behavior until the force value of 1500N was the initial stability of each sample so the real elastic behavior was considered after this value. The slope of each curve was calculated to identify the experimental rigidity of each sample.

Figure 37: Displacement sensor position.

Figure 38: wood to wood connection in test.

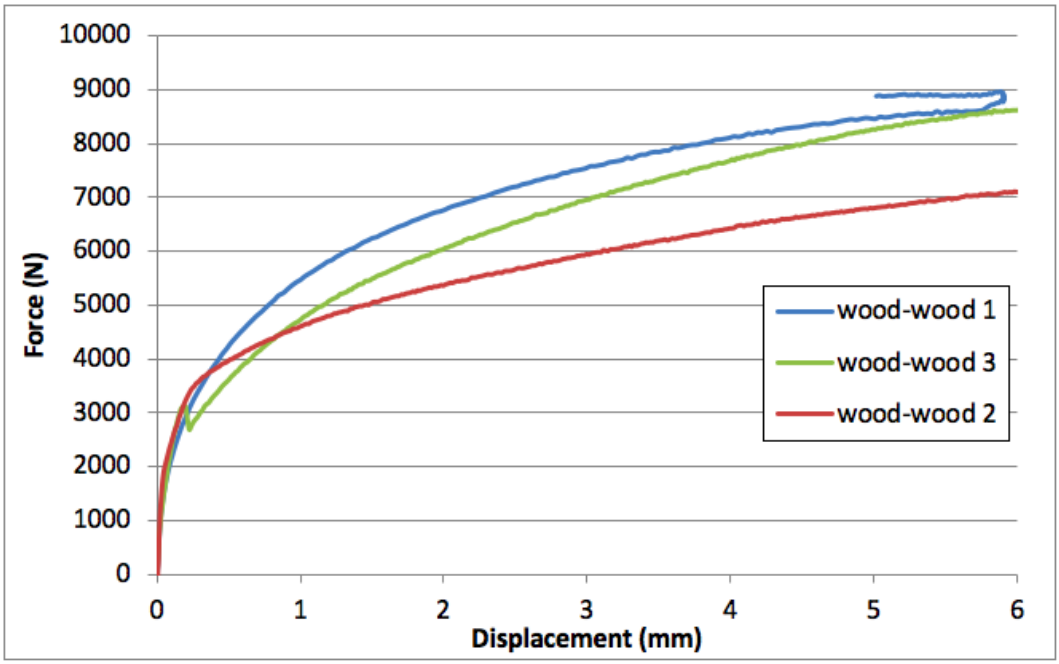


Figure 39: Force displacement diagram of wood to wood connections.

The Theoretical rigidity was calculated according to the Eurocode 5 as below:

$$K_{ser,theo} = \frac{\sqrt{\rho_{m1} \cdot \rho_{m2}}^{1.5} \cdot d}{23} \cdot n = \frac{\sqrt{640.420}^{1.5} \cdot 3}{23} \cdot 4 = 6159.2\ kN/m$$

The average rigidity of the first sample Kexp = **6876 KN/m** , The average rigidity of the second sample Kexp = **8981 KN/m** , The average rigidity of the third sample Kexp = **2911 KN/m** So the average rigidity of the wood to wood connection can be calculated as

Total Average rigidity K_{exp} of wood wood assembly = (2911 + 8981 + 6876)/ 3 = 6256 KN/m

- *Wood – steel connection*

(date: 08/07/2020)

Three samples of wood to steel connection were prepared at the laboratory. In each sample 2 bolts M8 were used to connect the wooden part with the steel part. Two displacement sensors were fixed at the two edges in the sample to investigate the displacement between the wooden latte and steel plate of the assembly, the total capacity of each sensor was 10 mm. A wooden foundation was fixed at the end of each sample for ensuring the stability of the sample during the test as illustrated in the figure 40 below. A speed test 2 mm / min was applied at each sample with the compression machine at the laboratory. The force displacement diagram was registered for each sample.

According to tests results in Figure 42 it was obvious that the real elastic behavior of the samples was starting after a value of 0.5 mm.

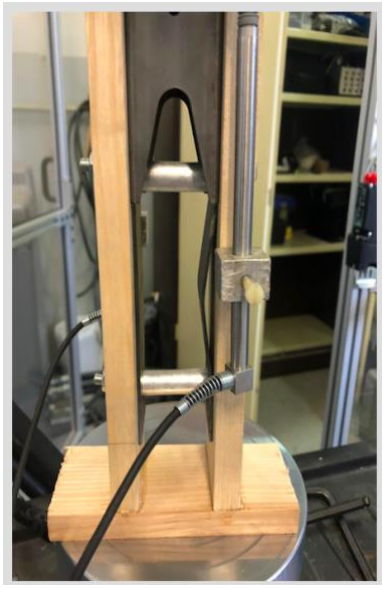


Figure 40: Displacement sensor position.

Figure 41: wood to steel connection in test.

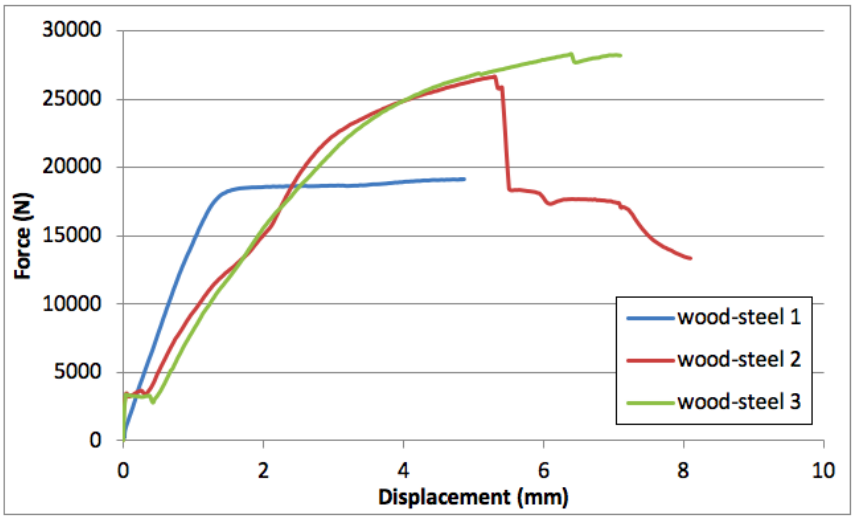


Figure 42: Force displacement diagram of wood to steel connections.

According to tests results of the wood to steel connections, it was obvious the elastic behavior of each sample was little bit different about the other. This diagram explain the same differences between the entire beam tested before. Indeed the configuration of the assembly and the sizes of holes in each part of assembly was not fitting the required dimensions. In addition, the spacers between the bolts were not with the same dimensions of the bolts. For sample no. 1 the slope of the curve was calculated directly, but for the sample no. 2 and 3 the slopes were considered after a value of 0.5 mm of the entire deflection.

The Theoretical rigidity was calculated according to the Eurocode 5 as below:

$$K_{ser,Bolts}\ \frac{\rho_m^{1.5}.d}{23} = \frac{640^{1.5}.5}{23}.\ 2.\ 2 = 14079\ kN/m$$

The average rigidity of the first sample Kexp = **14699 KN/m** , The average rigidity of the second sample Kexp = **87796 KN/m** , The average rigidity of the third sample Kexp = **8483 KN/m.**

So the average rigidity of the wood to steel connection can be calculated as:

Total Average rigidity K_{exp} of wood steel assembly $K_{exp} = \frac{14699+8796+8483}{3} = 10659$ KN/m

- *Steel – steel connection (flexural test)*

(date: 22/07/2020)

As the middle part of the assembly is under flexural behavior, the experimental tests that considered in order to determine the rigidity was 4 point flexural tests (rigidity in rotation). Two samples of steel to steel connection with length 38 cm were prepared at the laboratory. Two tubes were welded at the two edges of the beam to avoid the behavior of shear. one displacement sensor was fixed at the middle of each sample to investigate the displacement of the assembly, the total capacity of the sensor was 10 mm. A speed test 1 mm / min was applied at each sample with the 4 points flexural test machine at the laboratory. The force-displacement diagram was registered for each sample. According to tests results in Figure 44 it was obvious that elastic behavior of the samples 1 and 2 were starting after a value of 0.4 mm and 0.7 mm respectively. The average rigidity of the assembly in rotation was around **23386 KN.mm**

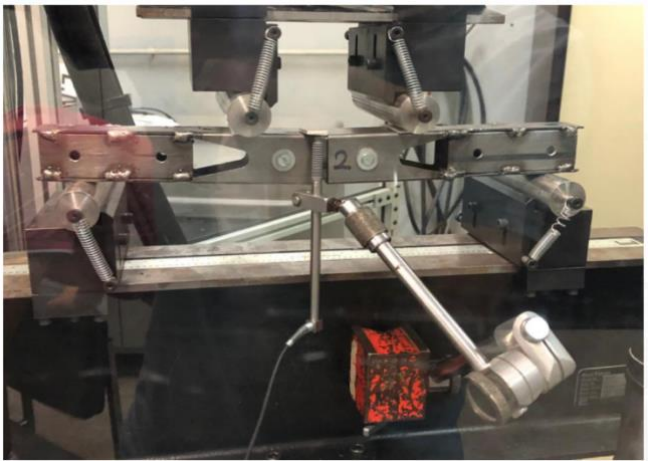


Figure 43: Displacement sensor position.

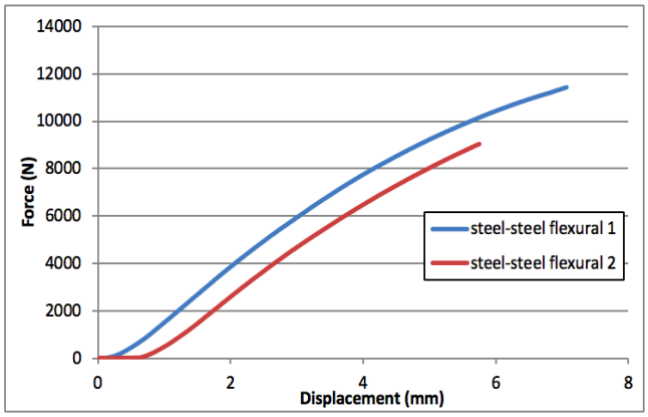


Figure 44: wood to steel connection in test.

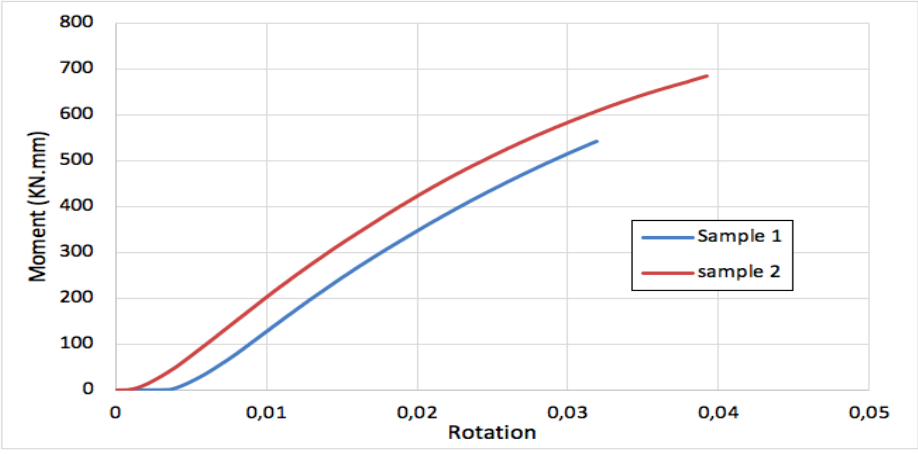
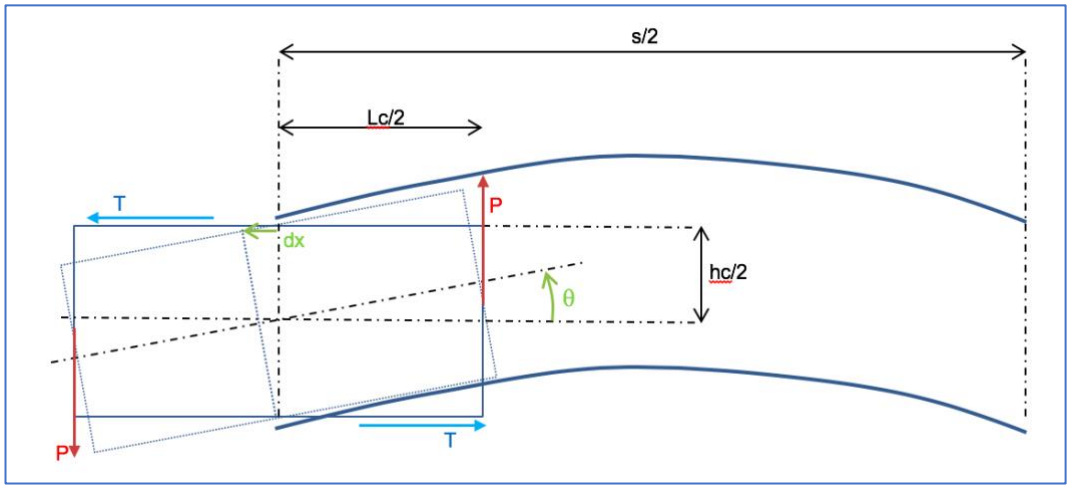


Figure 45: Moment rotation diagram of steel to steel connections.

Tilting Rigidity (Basculement)

The tilting rigidity of the shear wedges should consider in the calculation of the total rigidity of the beam. The calculation of the tilting indeed depends on the dimensions of the shear wedges and the distance center to center between shear wedges in the entire beam. In the tested beam, the length of the shear wedge was 10 cm and the distance center to center between shear wedges was 30 cm. The figure below illustrated the geometry of the shear wedge in loaded case of the structure and the method of calculation.



$$\frac{T}{Lc \cdot e} = \frac{P}{hc \cdot e}$$ then $$P = T \frac{hc}{Lc}$$

With the assumption of simply supported beam, we can calculate the rotation at the support by:

$$\theta = \frac{P \cdot \frac{Lc}{2} \cdot \left(\frac{s}{2} - \frac{Lc}{2}\right) \cdot \left(\frac{s}{2} + \frac{s}{2} - \frac{Lc}{2}\right)}{6 \cdot \left(\frac{s}{2}\right) \cdot E_{mean} \cdot \left(2 \cdot \frac{b \cdot e^3}{12}\right)}$$

where

S: distance between two shear blocks (cales) center to center.

Lc: The length of the shear block.

hc : the height of the shear block (distance between two lames)

b : length of the lame.

e: thickness of the latte.

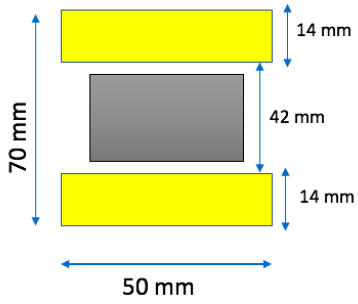
$$\theta = \tan^{-1}\left(\frac{dx}{hc}\right) \approx \frac{dx}{hc} \rightarrow dx = \theta.hc$$

Then the rigidity can be estimated by

$$K_{u,E,bascule} = \frac{T}{dx} = \frac{T}{\theta.hc}$$

$$= T.\left(\frac{3.s.E_{mean}.\left(\frac{b.e^3}{6}\right)}{Lc\ (s\quad Lc\backslash\ (\quad Lc\backslash}\right).\left(\frac{1}{hc}\right)$$

$$K_{u,E,bascule} = T.\left(\frac{s.E_{mean}.\frac{b.e^3}{2}}{\left(T.\frac{hc}{Lc}\right).\frac{Lc}{2}.\left(\frac{s}{2}-\frac{Lc}{2}\right).\left(s-\frac{Lc}{2}\right)}\right).\left(\frac{1}{hc}\right)$$



$$K_{u,E,bascule} = \frac{s.E_{mean}.b.e^3}{hc^2.\left(\frac{s}{2}-\frac{Lc}{2}\right).\left(s-\frac{Lc}{2}\right)}$$

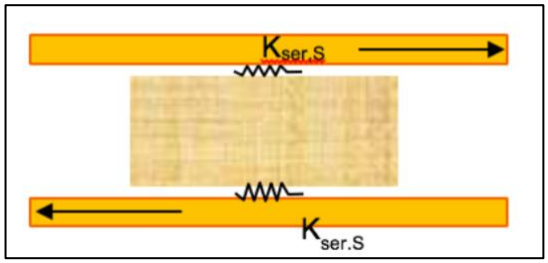
The estimation of the tilting for the tested beam can be express as:

$$K_{u,E,bascule} = \frac{300\ x\ 11000\ x\ 50\ x\ 14^3}{42^2.\left(\frac{300}{2}-\frac{100}{2}\right)x\left(300\ -\frac{100}{2}\right)}$$

$$\boxed{K_{u,E,bascule} = 10266\ KN/m}$$

Combination of rigidity

After identifying the rigidity of each part in the assembly, it will be possible to make the combination of the rigidity in order to express the global rigidity of the entire beam with the assembly. The combination will carry out between the rigidity of shear wedges in shear effects and the tilting rigidity, then the equivalent results will also be in combination with the rigidity of the wood steel connection under shear effects.



$$K_{u,\text{Bascul},CI} = \left[\frac{1}{\frac{1}{K_{u,O,\text{top},CI}} + \frac{1}{2.K_{u,BASC,CI}}} = \frac{1}{\frac{1}{K_{u,O,\text{bottom},CI}} + \frac{1}{2.K_{u,BASC,CI}}} \right]$$

$$K_{u,\text{Bascul},CI} = \left[\frac{1}{\frac{1}{3128} + \frac{1}{2\ x\ 10266}} = \frac{1}{\frac{1}{3128} + \frac{1}{2\ x\ 10266}} \right]$$
$$= 2715\ kN/m$$

$K_{u,\text{Bascul},CI}$ = 2715 x 6 Cales = 16290 KN/m

The total rigidity of the wood to steel connection in the entire beam is

$K_{u,\ assem\ b.a}$ = 10659 x 2 assemblages = 21318 KN/m

$$K_{u,\text{Bascul},CI,\ assem\ b.a} = \left[\frac{1}{\frac{1}{K_{u,O,\text{top},assem\ b.a}} + \frac{1}{K_{u,BASC,CI}}} = \frac{1}{\frac{1}{K_{u,O,\text{bottom},assem\ b.a}} + \frac{1}{K_{u,BASC,CI}}} \right]$$

$$K_{u,\text{Bascul},CI,\ assem\ b.a} = \left[\frac{1}{\frac{1}{10659} + \frac{1}{16290}} \right] = 6443\ \text{KN/m}\ \text{(for the entire beam)}$$

Equivalent moment of ineria

In the framework of a beam composed of 2 rectangular members e x b spaced by a wedge height hc, the equivalent inertia I_{eq} according to Eurocode 5 is expressed as the following way:

$$I_{z,eq} = \sum \gamma . a^2 . A_{latte} + \sum I_{z,latte}$$

Where

The area of the section of a single latte $A_{latte} = e.b$

The bending inertia of a single latte $I_{z,latte} = \frac{b.e^3}{12}$

The coefficient of efficiency on the part of Huygens γ

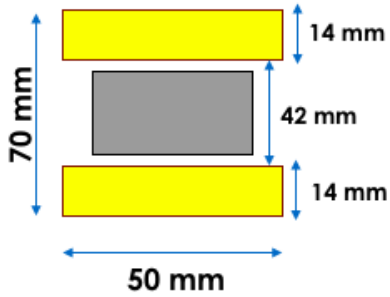
In order to calculate the equivalent rigidity, the coefficient γ should calculated.

$$\gamma_1 = \frac{1}{1+ k_1} \quad \text{with} \quad k_1 = \frac{\pi^2}{L^2} \frac{E_1 A_1}{\text{Ku}}$$

$$k_1 = \frac{\pi^2}{(2{,}4)^2} \frac{700 . 10^7 . 10^{-6}}{6443} = 1.861$$

$$\gamma_1 = \frac{1}{1+ k_1} = \frac{1}{1+1.861}$$

$$\gamma_1 = 0.349$$



$$E\, I_{theo} = E_1\, I_1 + E_2\, I_2 + \frac{E_1 A_1\ \gamma_1\ a^2}{1+ \gamma_1 \frac{E_1 A_1}{E_2 A_2}}$$

$$I_{eq} = 2\, I_1 + \frac{A_1\ \gamma_1\ a^2}{1 + \gamma_1}$$

$$I_{eq} = 2 \frac{5 .\ 1{,}4^3}{12} + \frac{7 .\ 0{,}349 .\ 5{,}6^2}{1 + 0{,}349}$$

- $I_{eq} = 59.07\ \text{cm}^4$

In the calculation of the inertia equivalent, it was considered that the top latte and the second latte are the same and have the same conditions. The formula takes into account the top and bottom of the latte section. The thickness of the latte considered was 14 mm and a width of 5à mm. Also the combination of the rigidities of shear wedges and the tilting and the steel to wood connection were included in the formula with a representative value of Ku.

Discussion and perspectives

The mechanical behavior of the wooden beams with and without assembly were studied in order to verify the global behavior of the gridshells beams. The results showed the approximate elastic behavior in all tested beam with different initial values of deflection before loading the beam. The values of initial deflections were varies from 7 to 10 mm in wooden beams only and from 33 mm to 53 mm for the wooden beams with assembly.

The strain gauges results proved the unsuitable top side of the assembly that is not working with the top latte of the wooden beam which means that the assembly need to redesign in order to ensure a complete contact between the steel part of the assembly with the wooden part of the beam.

The characterization of each part of the assembly with the shear wedges were identified in estimate the rigidity of each part in order to predict the total rigidity of the beam. Also the equivalent rigidity was taken into account in order to identify the equivalent moment of inertia of the entire beam.

The results of the preliminary tests showed that only two screws on each shear wedge are not enough according to the mode of failure appeared on the results

All the beams tested were in one plan of loads, and normally, in the real structure of Gridshells the beam will be loaded on two plans, these protocol may effect on the final results.